Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (OLD) EXAMINATION – SUMMER-2019

Subject Code: 110008 **Date: 06/06/2019**

Subject Name: Maths - I

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Evaluate : $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$ **02**

(ii) Evaluate : $\lim_{x\to 0}\left(\frac{1}{x}\right)^{1-\cos x}$ **02**

(iii) Find Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for functions $u = x\sin y, v = y\sin x$. **03**

(b) (i) Sketch the region and find the area bounded by ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. **03**

(ii) Using Lagrange's mean value theorem, prove that **04**
$\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}$

Q.2 **(a)** (i) Verify Rolle's theorem for $f(x) = x(x+3)e^{-\frac{x}{2}}$ in $-3 \le x \le 0$. **04**

(ii) Find two non-negative numbers whose sum is 9 such that the product of one number and the square of the other is maximum. **03**

(b) (i) Prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \frac{1}{2}\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots\right)$. **04**

(ii) Find the absolute maximum and minimum values of $f(x) = \frac{x^3}{x+2}$ in interval $[-1, 1]$. **03**

Q.3 **(a)** (i) Test the convergence of the series: $\sum_{n=1}^{\infty}\frac{n^n x^n}{(n+1)^n}, x > 0$. **04**

(ii) Test the convergence of the series: $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}$ **03**

(b) (i) Test the convergence of the series: $\frac{1}{2}-\frac{2}{5}+\frac{3}{10}-\frac{4}{17}+\cdots$ **04**

(ii) Test the convergence of the series: $\sum_{n=1}^{\infty}\frac{n!}{n^n}$ **03**

Q.4 **(a)** (i) Find extreme values of $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7$ **04**

(ii) Find all first and first and second order partial derivatives for $f(x,y) = x^2\sin y + y^2\cos x$. Hence, verify mixed derivative Theorem. **03**

(b) (i) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$; show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u\cos 3u$. **04**

(ii) If $u = f(x-y, y-z, z-x)$ then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$. **03**

Q.5 **(a)** Sketch the region of integration and evaluate by reversing the order of Integration for integral $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ **07**

(b) (i) Evaluate the integral $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}dxdy$ by changing into polar coordinates. **04**

(ii) Evaluate $\int_0^1\int_0^{1-x}\int_0^{x+y} e^z dxdydz.$ **03**

Q.6 **(a)** (i) Find directional derivative of $\emptyset = xy^2 + yz^2$ at point $(2,-1,1)$ in the direction of vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. **04**

(ii) If $\bar{r} = t^3\hat{\imath} + (2t^3 - \frac{1}{5t^2})\hat{\jmath}$, then show that $\bar{r} \times \frac{d\bar{r}}{dt} = \hat{k}$. **03**

(b) Evaluate $\int_C \bar{F}\cdot d\bar{r}$, where, $\bar{F} = (x^2+y^2)\hat{\imath} - 2xy\hat{\jmath}$. Where, C is the rectangle in XY-plane bounded by $y = 0, x = a, y = b, x = 0$. **07**

Q.7 **(a)** Verify Green's theorem for $\oint_C[(x^2-2xy)dx + (x^2y+3)dy]$. Where, C is the boundary of the region bounded by $y = x^2$ and the line $y = x$. **07**

(b) (i) Show that $\bar{F} = (y^2 - z^2 + 3xy - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational. **04**

(ii) Evaluate $\oint_C[e^x dx + 2ydy - dz]$ by Stoke's theorem. where, C is the curve $x^2 + y^2 = 4, z = 2$. **03**

****